MODIFIED ALGEBRAIC APPROACH

$$a\,Zn + b\,HNO_3 + c\,NO \longrightarrow d\,Zn(NO_3)_2 + e\,NH_3$$

$\therefore$ Zn: $a = d$ N: $b + c = 2d + e$

H: $b = 3e$ O: $3b + c = 6d$

- BE ABLE TO WRITE DOWN SUCH RELATIONSHIPS

- THEREFORE 4 EQUATIONS AND 5 UNKNOWNS $\Rightarrow$ SOLUTION IS NOT UNIQUE THOUGH LOWEST SET OF INTEGERS IS

- SINCE SOLUTION IS NOT UNIQUE AND WE ARBITRARILY SET A COEFFICIENT TO 1 I FIND IT EASIER TO DO THIS WITH ORIGINAL SET OF EQUATIONS (E.G., $a = c = 1$ ON RIGHT)

LET $a = d = 1$ THEN $b = 3e \Rightarrow$ (N, O EQN BECOME)

$$\left.\begin{aligned} 3e + c &= 2a + e \\ 3(3e) + c &= 6a \end{aligned}\right\} \Rightarrow \begin{aligned} 2e + c &= 2a \\ 9e + c &= 6a \end{aligned}$$

$$7e = 4a;\ e = 4/7$$

$$2(4/7) + c = 2;\ c = 6/7$$

$a : b : c : d : e$

$1 : 12/7 : 6/7 : 1 : 4/7 \Rightarrow 7 : 12 : 6 : 7 : 4$

Algebraic Method for Balancing Equations

BW 10/86, 9/91, 7/95

(use this only for DIFFICULT equations)

Appraisal of difficulty:
- H in 3 formulas
- O in 3 formulas
- N in 2 formulas
- I in 2 formulas

Balance this equation: $HNO_2 + HI \longrightarrow NO + I_2 + H_2O$

Assign an algebraic unknown (letter) as each coefficient:

$$a\,HNO_2 + b\,HI \longrightarrow c\,NO + d\,I_2 + e\,H_2O$$

These coefficients must be related because of mass balance and non-destruction of atoms in ordinary chemical reactions.

N : $a = c$

H : $a + b = 2e$

O : $2a = c + e$

I : $b = 2d$

charge : neutral = neutral

Instead of changing coefficients that were previously changed, this method solves simultaneously for all coefficients at once.

Often, this set of equations is easier to solve than the usual high school algebra examples for simultaneous equations.

But only use this algebraic method when you must, because algebra errors in lengthy calculations are common. If you perform unnecessary calculations and do not find the correct answer, it is likely to adversely affect your exam scores.

- Now we have four independent equations with five unknown variables. We also have the knowledge that all five unknowns (coefficients a, b, c, d, & e) must be integers.

To solve: e is in only two equations. Solve the O equation for e, and substitute its value into the H eq:

O: $e = 2a - c$

into H eq: $a + b = 2(2a - c)$

$a + b = 4a - 2c$

c=a by N eq: $a + b = 4a - 2a$

$a + b = 2a$

subtr. b from both sides, $\therefore b = a$

$b = \underbrace{a = c}_{\text{(by N eq)}} = \overbrace{2d}^{\text{(by I eq)}}$

H eq: $a + b = 2e$

but b=a, so $2a = 2e$, $\therefore a = e$

$$\boxed{b = a = c = 2d = e}$$

To solve for the 5 integers, we can use the method of **simultaneous equations** from algebra.

Such problems can also be solved by Cramer's rule (a matrix method) or by other linear algebra methods. See J. Chem. Educ., 6/85, p.507-508. or Chemistry magazine, 3/75, p.19-21.

A Challenge: Write a computer program to balance chemical equations.

At this point we can find a possible set of integers by finding the LCM (lowest common multiple) of the multipliers in the equation above. Since d has the highest multiplier, d must be the smallest coefficient in the balanced equation, so we try d = 1. Then $a = 2$, $b = 2$, $c = 2$, and $e = 2$

$$\boxed{2HNO_2 + 2HI \longrightarrow 2NO + I_2 + 2H_2O}$$

Check balance:
- H: $2 + 2 = 2(2)$ ✓
- O: $2(2) = 2 + 2$ ✓
- N: $2 = 2$ ✓
- I: $2 = 2$ ✓

Practise the algebraic method by balancing these as-yet unbalanced equations:

$I_2 + HNO_3 \longrightarrow HIO_3 + NO_2 + H_2O$

$BF_3 + H_2O \longrightarrow H_3BO_3 + HBF_4$

$P + HNO_3 + H_2O \longrightarrow NO + H_3PO_4$

$Al(NO_3)_3 + NH_3 + H_2O \longrightarrow Al(OH)_3 + NH_4NO_3$

$Zn + HNO_3 \longrightarrow Zn(NO_3)_2 + NH_3 + H_2O$

$CaSO_4 + CH_4 + CO_2 \longrightarrow CaCO_3 + S + H_2O$

$NH_4NO_3 + C_{12}H_{26} \longrightarrow CO_2 + H_2O + N_2$

Other Problems:

Balance the following equations:

	Equation	Description ~~Importance~~ of this equation
a.)	$CO_2 + H_2O \rightarrow C_6H_{12}O_6 + O_2$ (glucose)	formation of sugars by photosynthesis (uses a chlorophyll catalyst).
b.)	$CO_2 + H_2 \rightarrow CH_4 + H_2O$	reaction in methane bacteria (producing swamp gas = CH_4).
c.)	$CO + H_2 \rightarrow C_8H_{18} + H_2O$ (octane)	a possible prebiotic synthesi of octane (a hydrocarbon) from carbon monoxide.
d.)	$Fe + O_2 \rightarrow Fe_2O_3$	the reaction which tells us why no free (elemental) iron is found on the earth's crust